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The nature of the absorbing-state phase transition in the diffusive epidemic process

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Abstract
In the diffusive epidemic process (DEP), particles of two species (A and B) hop on a lattice and undergo reactions B → A and A + B → 2B; the B-free state is absorbing. Renormalization group analysis predicts a continuous phase transition to the absorbing state when the hopping rate of B particles, D_B, is greater than or equal to that of A particles, and a discontinuous transition for $D_A > D_B$. Monte Carlo simulations of the one-dimensional DEP suggest that, on the contrary, the transition is continuous in all cases. Here we present strong evidence for a continuous transition for $D_A > D_B$ in the two-dimensional model as well. Our results suggest that hysteresis is absent in both the one- and two-dimensional cases.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The diffusive epidemic process (DEP) [1] is a nonequilibrium model in which two kinds of particles, A and B, diffuse on a lattice and undergo reactions B → A and A + B → 2B. There is no intrinsic limit on the number of particles that may be present at a given site; the total number of particles is conserved. A natural interpretation of the DEP has A particles representing healthy organisms and B's infected ones, with the reactions above corresponding, respectively, to spontaneous recovery, and transmission of disease on contact. The DEP exhibits a phase transition to an absorbing state [2–5], in which all particles are of type A. Absorbing-state transitions arise in many models of epidemics, population dynamics and autocatalytic chemical reactions, and have attracted much interest in nonequilibrium statistical mechanics, in efforts to characterize the associated universality classes. The simplest example is the contact process (CP) [2, 6] or its synchronous update version, directed percolation (DP).

Critical scaling at absorbing-state phase transitions has been studied extensively, both theoretically and numerically. A central conclusion is that the critical behavior of the DP type is generic for models exhibiting a continuous phase transition to an absorbing state, in the absence of any additional symmetries or conserved quantities [7, 8]. In particular, allowing particles to diffuse at a finite rate does not affect the critical behavior of the contact process [9].

The DEP offers a more complicated scaling scenario than that of DP. Let D_A (D_B) denote the diffusion rate of A (B) particles. The absorbing-state phase transition in the DEP appears to belong to three distinct universality classes, depending on whether $D_A < D_B$, $D_A = D_B$, or $D_A > D_B$ [10]. Simulations of the one-dimensional model [11] support this scenario. It should be noted that all three classes are distinct from that of DP.

The DEP was initially studied via renormalization group (RG) methods [1, 10, 12]. The analysis of Oerding *et al* [12] predicts a continuous phase transition in the first two cases, but for $D_A > D_B$ it yields a discontinuous transition. These authors argue that this is an example of a *fluctuation-induced* first-order transition, and provide some numerical evidence of this in two dimensions. Numerical simulations for equal diffusion rates [13] yielded results in disagreement with the RG prediction $\nu_\perp = 2$ (see [14, 15]). Subsequently, the simulations reported in [16] appeared to resolve this point, but suggested other departures from RG predictions. In particular, simulations of the one-dimensional model show a continuous phase transition in all three cases [11, 16]. In the present work we focus on the case $D_A > D_B$; our results strongly suggest that the transition is in fact continuous in two dimensions as well. We also revisit the one-dimensional model, checking for signs of hysteresis.

The DEP is defined on a lattice of L^d sites. A configuration is specified by the set of variables a_j and b_j, denoting the number of A and B particles at each site j. The model is a continuous-time Markov process characterized by four kinds of transitions:

- Hopping of A particles to a randomly chosen nearest neighbor (NN) site, at rate D_A.
- Hopping of B particles to a randomly chosen NN site, at rate D_B.
- Transformation of B particles to A particles, at rate r.
- Transformation of A particles to B particles, in the presence of a B particle at the same site, at a rate of λ per A–B pair.

This means that a given site j loses (via diffusion) an A particle at rate $D_A a_j$ (similarly for loss of a B particle), undergoes the process B $\to$ A at rate rb_j, and the process A + B $\to$ 2B at rate $\lambda a_j b_j$. Note that all transitions conserve the total particle number $N = \sum_j (a_j + b_j)$. The process is characterized by the parameters D_A, D_B, r, λ, and the particle density $\rho = N/L^d$. Since one of the rates may be eliminated through a suitable scaling of time, we set $\lambda = 1$ from here on. We study the DEP in Monte Carlo simulations, using an algorithm that faithfully reflects the above transition rates [11].

Before presenting, in the sections that follow, quantitative evidence for a continuous transition when $D_A > D_B$, we consider some qualitative aspects of the problem. The usual argument for a discontinuous transition to an absorbing state depends on showing that an active state in which the order parameter is small is unstable, so that the order parameter does not tend to zero continuously as the relevant control parameter is varied. Consider, for example, the triplet creation model [17–19], in which particles hop at rate D and undergo reactions A $\to$ 0 and 3A $\to$ 4A. (In one dimension, a set of three consecutive occupied sites is required for particle creation.) One argues that for high diffusion rates, when the particle density is small, triplets are rare and the system falls into the particle-free absorbing state, whereas survival is possible if the particle density is sufficiently high. In the CP, by contrast, only a single particle is needed to create another one, and the transition is continuous. In the DEP, creating a B particle requires one B and one A, so there is no obvious reason for a state with a low density of B's to be unstable. Oerding *et al* [12] nevertheless argue that due to fluctuations in ρ_A, a given region will from time to time experience a subcritical density value and lose all its B particles. If B's from surrounding active regions cannot diffuse in rapidly enough to repopulate the B-free region, then this species will go extinct, leading to a fluctuation-induced discontinuous transition. These authors arrive at an estimate for the discontinuity in the B-particle density $\Delta\rho_B$ at the transition; for $D_B \lesssim D_A$,

$$\Delta\rho_B \propto (4-d)\left(1-\frac{D_B}{D_A}\right)^{2/(4-d)} \qquad (1)$$

in $d < 4$ dimensions.

The search for a discontinuous transition in [12] is motivated by the fact [10] that the RG recurrence relations flow, for $D_A > D_B$, to an unphysical region of parameter space. While the argument for a fluctuation-induced discontinuous transition seems quite plausible, there are many points where the intuitive picture outlined above might go wrong. In the one-dimensional case, for example, we find that regions with $\rho_B > 0$ are associated with a *lower than average* value of ρ_A, not a higher than average value as assumed in [12]. Oerding *et al* complemented their theoretical arguments with simulation results apparently showing a hysteresis loop in the order parameter as a function of the particle density. We comment on the apparent conflict between our simulation results and those of [12] in section 3.

It is interesting to consider for a moment the limit $D_A \to \infty$, with all other rates of order unity. In this limit, there are infinitely many A particle hopping events between any events involving hopping of a B particle, infection or recuperation, so the A particles are uniformly distributed over the lattice. The probability of having exactly a_j particles at site j follows a Poisson distribution with parameter $\rho_A = \rho - \rho_B$. The rates of infection, recovery and diffusion of the B particles are independent of their positions, so that the process is characterized by the total number N_B of Bs. N_B is a Markov process with transition rates $W(N_B \to N_B{-}1) = rN_B$ and $W(N_B \to N_B{+}1) = N_B\rho(1-\rho_B/\rho)$, that is, a contact process on a complete graph of $N = \rho L^d$ sites, with creation rate ρ and annihilation rate r. The latter process is known to suffer a continuous, mean-field-like transition to the absorbing state at $\rho = r$. Thus, if the transition were discontinuous for $D_A > D_B$, it would have to revert to being continuous as $D_A \to \infty$. (One might expect the transition to be continuous for large but finite D_A as well.)

Finally, we note that the one- and two-site cluster approximations predict a continuous transition in the DEP regardless of the relative values of D_A and D_B [11]. (Note that these approximations are in fact sensitive to the diffusion rates.) While it is not uncommon for cluster approximations to predict that a transition is discontinuous when it is in fact continuous (as in the case of Schlögl's second model [8, 20]), we are not aware of examples in which both the one- and two-site approximations incorrectly predict a *continuous* transition.

The foregoing arguments, while lacking rigor, make it quite plausible that the DEP exhibits only a continuous transition. To provide clearer evidence on this issue, we perform simulations probing several aspects of the phase diagram and scaling behavior. The remainder of this paper is organized as follows. In section 2 we report results of quasistationary simulations of the two-dimensional model. Section 3 is devoted to hysteresis studies and section 4 to an analysis of the evolution starting from very different initial values of the order parameter. In section 5 we summarize our findings and discuss them in the light of earlier studies.

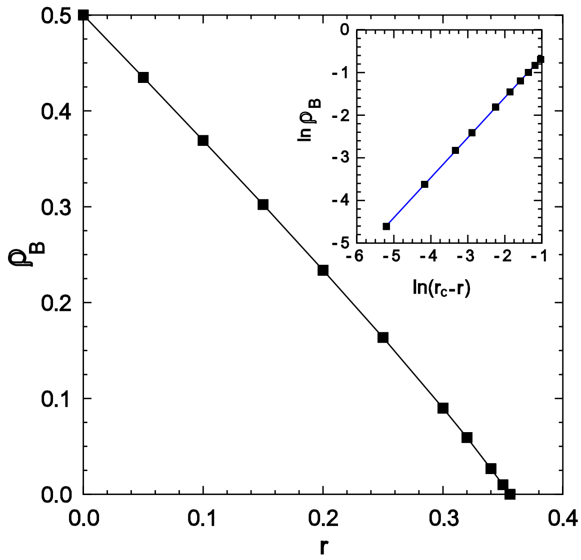

Figure 1. Order parameter ρ_B versus recovery rate for $D_A = 4$, $D_B = 1$, and $\rho = 0.5$ in the two-dimensional DEP. The point on the r axis represents our best estimate for the critical value, $r_c = 0.3555(5)$. Inset: ρ_B versus $r_c - r$ on log scales. The slope of the straight line is 0.94.

2. DEP in two dimensions: quasistationary simulations

We begin our discussion of the two-dimensional DEP with the results of quasi-stationary (QS) simulations. This method determines properties conditioned on survival, at long times (the QS regime). For details and examples see [21]. In [11], we showed that QS simulations are capable of discriminating between a continuous and a discontinuous absorbing-state phase transition.

To ensure that A particles diffuse at a substantially higher rate than Bs, we use $D_A = 4$ and $D_B = 1$. In the QS regime, we accumulate histograms of the time during which the system has exactly 1, 2, . . . n, . . . , B particles. The histogram is used to calculate the QS order parameter ρ_B and the moment ratio $m = \langle \rho_B^2 \rangle / \rho_B^2$. The lifetime τ is given by the mean time between attempted visits to the absorbing state, in the QS regime. We study ρ_B, τ and m as functions of the recovery rate r, at fixed total density ρ, for both $\rho = 1$ and $\rho = 1/2$.

The model is simulated on square lattices of $L \times L$ sites (with periodic boundaries), with $L = 20, 40, 80, \ldots, 640$. Results are typically obtained from averages over 5 to 10 independent realizations, each lasting 10^6 time units, with an initial portion (10^4 to 10^5 time steps, depending on the system size), discarded to ensure the system has attained the QS state. [The time increment associated with a given event is $\Delta t = [N_A D_A + (N_B + r) D_B + \Sigma_{AB}]^{-1}$, where $\Sigma_{AB} = \sum_i a_i b_i$ is the total number of reactive A–B pairs. The number of saved configurations used in the QS simulation procedure [21] ranges from 100, for $L = 20$, to 10, for $L = 640$. Values of the replacement probability p_{rep} range from 10^{-3} to 10^{-4} (smaller values for larger L).] Initially, half of the particles are of type A and half of type B; the particles are distributed randomly and independently over the sites, so that the distribution of a and b at a given site is essentially Poissonian.

Figure 1 shows our result for the order parameter, ρ_B, for total density $\rho = 0.5$. (The results for $\rho = 1$ are qualitatively similar.) In this graph we only plot values for which finite-size effects are absent; for example, our results for $r = 0.35$ are $\rho_B = 0.009\,94(5)$ and

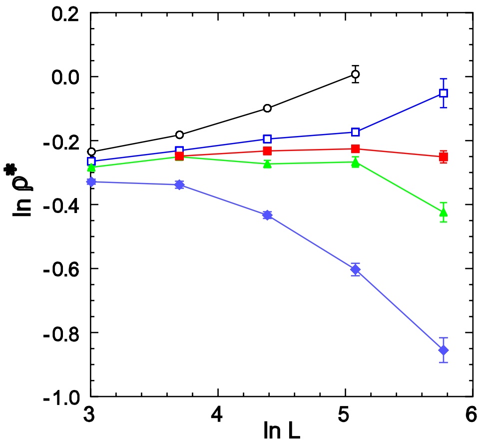

Figure 2. The scaled order parameter $\rho_B^* = L^{\beta/\nu_\perp}\rho_B$ versus system size for $D_A = 4$, $D_B = 1$ and $\rho = 1$ in the two-dimensional DEP, $\beta/\nu_\perp = 0.885$. Recovery rates $r = 0.735$, 0.738, 0.739, 0.740 and 0.745 (upper to lower).

0.009 98(10) for $L = 320$ and 640, respectively. The results strongly suggest that $\rho_b \to 0$ continuously as r tends to r_c; the data effectively exclude a gap in ρ_B larger than about 0.01. The finite-size scaling analysis described below yields a critical recovery rate of $r_c = 0.3555(5)$ for $\rho = 0.5$. The inset of figure 1 is a plot of ρ_B versus $r_c - r$ on log scales, showing power-law behavior with an exponent $\beta = 0.94(2)$. (Interestingly, the value for β in the corresponding one-dimensional case [11] is 0.93(14). Thus in both one and two dimensions, β is quite close to its mean-field value of unity for $D_A > D_B$.)

As in the one-dimensional case, the argument for a continuous transition is greatly strengthened by our observation of critical scaling. We determine the critical recovery rate $r_c(\rho, D_A, D_B)$ using the criteria of power-law dependence of ρ_B and τ on system size L (i.e., the usual finite-size scaling relations $\rho_B \sim L^{-\beta/\nu_\perp}$ and $\tau \sim L^z$). The moment ratio m is also a sensitive indicator of criticality, as it increases (decreases) sharply with system size L, for $r > r_c$ ($r < r_c$).

Using our best estimate, $\beta/\nu_\perp = 0.885(10)$, we plot the scaled order parameter $\rho_B^* = L^{\beta/\nu_\perp}\rho_B$ versus system size in figure 2. The data for $r \leqslant 0.738$ curve upward, while those for $r = 0.740$ curve downward, leading to the estimate $r_c = 0.7390(5)$. Similarly, the data for the QS lifetime τ (see figure 3) show power-law scaling for $r \simeq r_c$, and significant curvature away from r_c. The moment ratio is plotted versus L^{-1} in figure 4, showing that at the critical point m appears to converge to a value of 1.30, while for off-critical values of r there is no apparent limit. (For $r < r_c$ one expects $m \to 1$ as $L \to \infty$ [22].)

The studies described above lead to estimates for the critical exponent ratios $\beta/\nu_\perp$, $z = \nu_\parallel/\nu_\perp$, and m, as summarized in table 1. As in [11], we use the relation

$$\left|\frac{\partial \ln \rho_B}{\partial r}\right|_{r_c} \propto L^{1/\nu_\perp}, \tag{2}$$

to estimate the exponent $\nu_\perp$. We stress that although the main purpose of the present study is not to determine critical exponents with precision, the clear evidence of scaling provides further support for a continuous phase transition in the DEP with $D_A > D_B$.

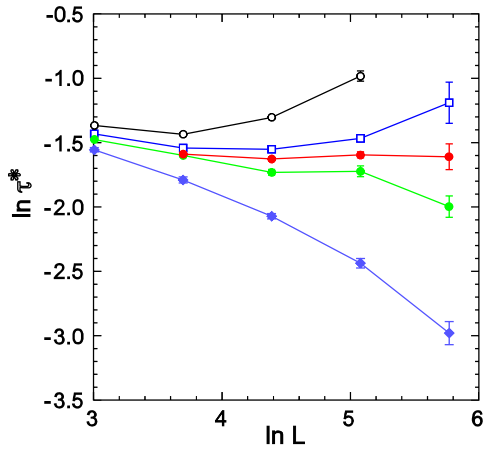

Figure 3. The scaled QS lifetime $\tau^* = L^{-z}\tau$ versus system size for $D_A = 4$, $D_B = 1$ and $\rho = 1$ in the two-dimensional DEP, $z = 1.9$. Recovery rates $r = 0.735$, 0.738, 0.739, 0.740 and 0.745 (upper to lower).

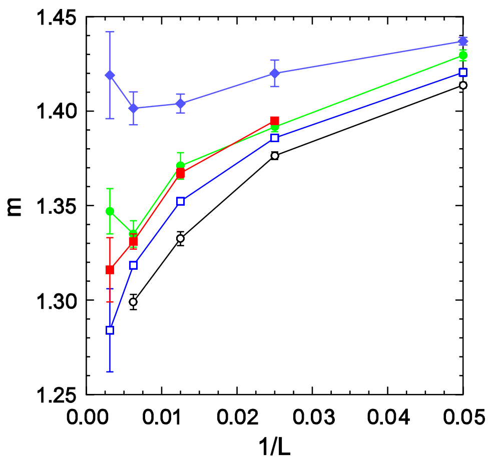

Figure 4. Moment ratio m versus $1/L$, for $D_A = 4$, $D_B = 1$ and $\rho = 1$ in the two-dimensional DEP. Recovery rates $r = 0.735$, 0.738, 0.739, 0.740 and 0.745 (lower to upper).

Table 1. Critical parameters of DEP in two dimensions, $D_A > D_B$.

ρ	r_c	$\beta/\nu_\perp$	z	β	$\nu_\perp$	m
0.5	0.3555(5)	0.87(4)	1.88(8)	0.94(2)	1.03(3)	1.34(3)
1.0	0.7390(5)	0.89(5)	1.90(8)	0.96(3)	1.10(4)	1.30(3)

3. Absence of hysteresis

A hallmark of a discontinuous transition is hysteresis: starting in a given phase (I), and slowly varying a control parameter, the system shows no immediate change on crossing a

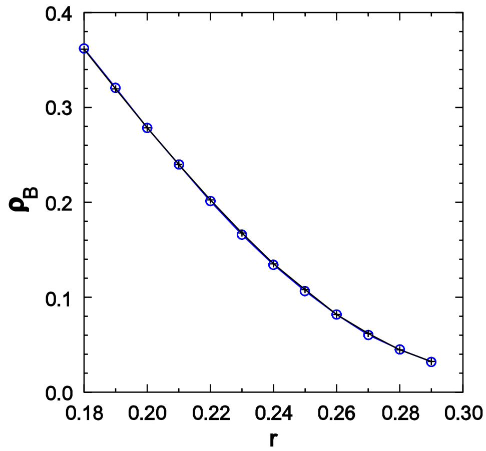

Figure 5. Order parameter versus recovery rate r in the one-dimensional DEP, $D_A = 0.5$, $D_B = 0.25$, $h = 10^{-5}$ and $L = 4000$. Symbols: +: increasing r; ○: decreasing r.

phase boundary, but rather tracks the metastable continuation of phase I, until a sufficiently strong fluctuation provokes a rapid switch to the (stable) phase (II). Reversing the process, the system tracks the metastable extension of phase II. This scenario, familiar from equilibrium studies, needs to be modified slightly when dealing with an absorbing-state phase transition, since the system cannot escape from the absorbing phase. Bideaux, Boccara and Chaté [23] showed that when the transition is discontinuous, adding a weak source of activity converts the absorbing phase to a *low-activity phase* (II), distinct from the high-activity phase (I). It is then possible to observe a hysteresis loop between phases I and II as one varies the relevant control parameter in the neighborhood of the transition. This approach has been used to demonstrate a discontinuous phase transition in a certain probabilistic cellular automaton [23] and in the triplet creation model [17]. (Another method for demonstrating hysteresis in models with an absorbing state is via *constant coverage* simulation [24, 25]).

In the case of the DEP, we impose a weak source of activity by adding to the set of transitions the process A → B at rate $h \ll 1$. (Each A particle is susceptible to this 'spontaneous activation', regardless of its position or the numbers of A and B particles at its site.) The B-free state is no longer absorbing and we may simulate the model directly, without recourse to the quasistationary method. In one dimension, using $h = 10^{-5}$ and rings of 2000 and 4000 sites, we search for evidence of hysteresis as we vary the parameter r. In these studies the particle density is 1, $D_A = 0.5$ and $D_B = 0.25$, while r ranges from 0.18 to 0.29, which includes the transition value $r_c = 0.2325(10)$ found in [11]. Starting from $r = 0.18$ and a random initial condition with $\rho_B = 0.5$, we allow the system to relax for a number τ of MC steps and then determine ρ_B over an interval of 4τ steps. Then r is increased by 0.01 and the evolution continues, using the final configuration at the previous r value as the initial configuration for the new one. This process is repeated using 5τ MC steps for each r value, with data for ρ_B collected over the final 4τ steps. Once the simulation at $r = 0.29$ is completed, we reverse the process reducing r by 0.01 at each stage. This procedure is repeated 125 times to obtain precise results for ρ_B over the cycle. Figure 5 shows a typical result for a ring of 4000 sites, using $\tau = 2.5 \times 10^5$. It is clear that $\rho_B(p)$ has the same value, regardless of whether r is increased from a small initial value, or decreased from a value above r_c. Studies on rings of 2000 sites, with τ ranging from 10^4 to 10^5, similarly show no hint of hysteresis.

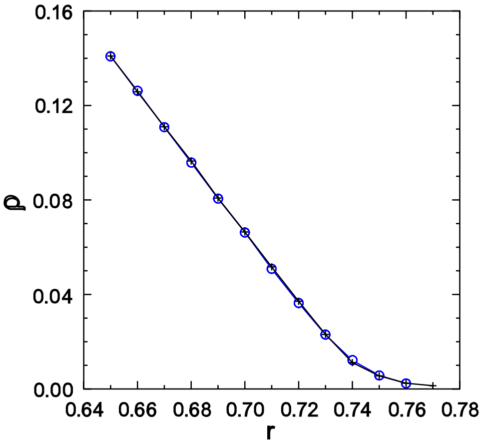

Figure 6. Order parameter ρ_B versus recovery rate r in the two-dimensional DEP, $D_A = 4$, $D_B = 1$, $h = 10^{-5}$, $L = 320$ and $\rho = 1$. Symbols: +: increasing r; ○: decreasing r.

We also searched for signs of hysteresis in the two-dimensional case. Using $D_A = 4$, $D_B = 1$, $h = 10^{-5}$ and 10^{-4}, and a particle density of unity, we find no evidence of hysteresis for system sizes $L = 80$, 160 and 320, and τ ranging from 10^3 to 10^4. Figure 6 shows a typical result. Note that as r is varied, the order parameter ρ_B varies smoothly from about 0.15 to $\approx 10^{-3}$. For $h = 10^{-5}$ and $r = 0.77 > r_c$, the stationary order parameter $\rho_B \to 1.4 \times 10^{-3}$ for large system sizes. Thus any discontinuity in ρ_B, if such were to exist, would need to have an amplitude $\leqslant 10^{-3}$. (In the one-dimensional case, our results imply an amplitude of $\leqslant 0.03$ or so.)

Our results would appear to be in conflict with those of [12] (see figure 2 of this reference), in which hysteresis is reported in a two-dimensional system, as the total density ρ is varied, at fixed infection and recovery rates, with $D_A/D_B = 8$. (Since there is a single transition line in the $\rho - r$ plane, the nature of the transition must be the same, varying r or ρ.) The simulations of Oerding *et al* in fact show the order parameter decreasing *continuously* to zero as $\rho \searrow \rho_c$. As ρ is increased, the graph of the order parameter versus ρ exhibits a flat portion, with $\rho_B \approx 0$; at some point (which varies considerably from cycle to cycle), ρ_B increases rapidly until it attains the value observed on the decreasing-ρ branch. Thus the hysteresis loop attributed to the DEP is triangular not rectangular (as observed for example in the triplet creation model, see figure 6 of [17]), and so is fully consistent with a continuous variation of ρ as a function of ρ or r. The flat portion of the loop observed in [12] is more likely a consequence of the very long time required to establish the quasi-stationary value, $\overline{\rho}_B$, starting from $\rho_B \ll \overline{\rho}_B$.

A related point is that a discontinuous transition and hysteresis are typically accompanied by a bimodal order parameter histogram. We verified in [11] that the histogram for the one-dimensional DEP is unimodal. We find the same to hold in two dimensions.

4. Initial-condition dependence as an indicator of a discontinuous transition

In this section we describe a simple alternative method for determining the nature (continuous or not) of a phase transition. The idea is that if the transition is discontinuous, then the evolution of the system depends strongly on the initial condition; conversely, at a continuous transition the evolution is toward the same quasistationary state, regardless of the initial condition.

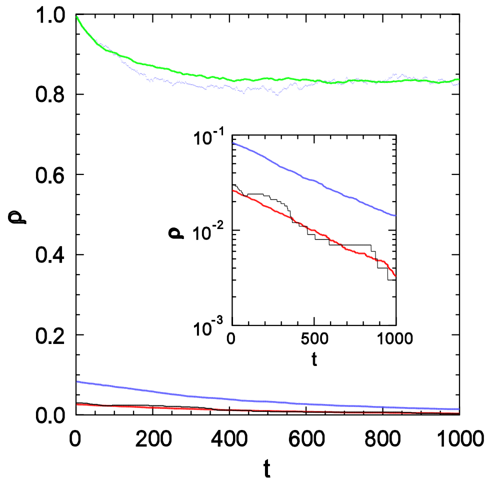

Figure 7. Evolution of the particle density in realizations of the TCM with $D = 0.98$, $\lambda = 9.60$. Bold lines: system size $L = 10^4$, initial densities (upper to lower) $\rho_i = 1$, 0.083 and 0.026; thin lines: $L = 10^3$, initial densities 1 and 0.03. Inset: semi-log plot of data in low-density regime.

To illustrate the method we turn to the one-dimensional triplet creation model (TCM), known to have a discontinuous transition at high diffusion rates [17–19]. In this process a particle (X) undergoes diffusion, annihilation (X → 0) and catalytic creation (3X → 4X), at rates D, $(1 - D)/(1 + \lambda)$ and $\lambda(1 - D)/(1 + \lambda)$, respectively. (Each site may be occupied by at most one particle; creation depends on the occupation of three successive sites.) We study the TCM at $D = 0.98$ where the transition, which occurs at reproduction rate $\lambda = 9.60$, is between a phase with a stationary particle density $\rho \simeq 0.83$, and the absorbing state, $\rho = 0$. Using systems of 10^3 and 10^4 sites, we find that for an initial particle density $\rho_i = 1$ the system evolves to the active state, while for $\rho_i \leqslant 0.3$ it rapidly approaches the absorbing state. (For $\rho_i < 1$ the initial particle positions are chosen at random over the sites of the lattice.) These results (figure 7) demonstrate that an active phase, characterized by a high value of the order parameter, is accessible starting from a high density, but not from a low one. This difference is evident for a rather modest system size ($L = 1000$ sites).

Now we turn to the diffusive epidemic process. In the one-dimensional case we find that the same stationary state is reached *independent* of the initial B-particle density $\rho_{B,i}$, even if this is very small, as expected at a continuous phase transition (see figure 8).

The results are similar in the two-dimensional case. Realizations with very different values of $\rho_{B,i}$ evolve to the same quasistationary state, as illustrated in figure 9. (We studied system sizes $L = 160$, 200 and 500, and several values of r, allowing for the possibility that the initial-density dependence might somehow be missed, working at a single recovery rate r.) The same picture applies in the subcritical regime; for $r > r_c$, realizations starting at both high and low B-particle densities rapidly approach the absorbing state.

The method used here seems quite effective in discriminating between continuous and discontinuous phase transitions, and involves a modest computational effort, as it does not require a large sample of independent measurements in the quasistationary regime.

5. Discussion

We apply various numerical approaches to the diffusive epidemic process with $D_A > D_B$ in one and two dimensions: quasi-stationary simulation, a search for hysteresis and a study

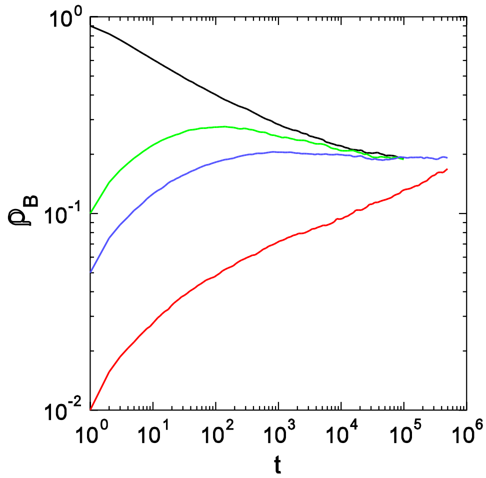

Figure 8. Order parameter versus time in the one-dimensional DEP, $L = 2000$, $D_A = 0.5$, $D_B = 0.25$, $\rho = 1$ and $r = 0.20$. From top to bottom, initial values $\rho_{B,i} = 0.9, 0.1, 0.05$ and 0.01.

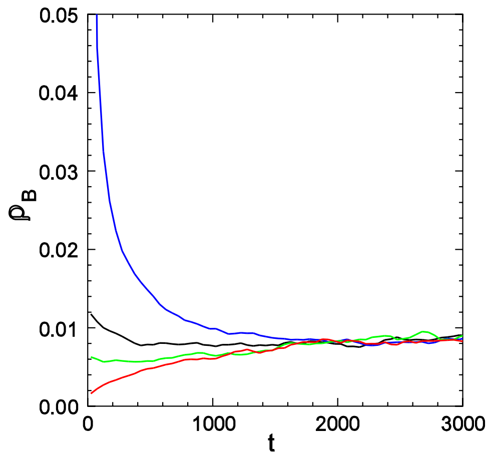

Figure 9. Evolution of the particle density in the two-dimensional DEP, $D_A = 4$, $D_B = 1$, $\rho = 1$, $r = r_c = 0.739$ and $L = 160$. Initial densities $\rho_{B,i} = 0.99, 0.01, 0.005$, and 0.001. Results are averages over surviving trials in a set of 250 realizations (1000 realizations for $\rho_{B,i} = 0.001$).

of the dependence on initial density. In no case do we find evidence of a discontinuous phase transition to the absorbing state. In fact, all our results are consistent with the scaling behavior (including finite-size scaling) expected at continuous transition to an absorbing state. In particular, we are able to limit the gap in the order parameter to a value of less than 0.001 in two dimensions and 0.03 in one dimension. Thus, any discontinuity in the order parameter would have to be extremely weak.

As was pointed out in section 2, the critical exponent β is quite close to its mean-field value of unity for $D_A > D_B$. In fact, for $D_A = 4$ and $D_B = 1$, the one-site mean-field approximation developed in [11] yields the critical values $r_c = 0.3726$ and 0.7655 for $\rho = 0.5$ and 1, respectively. These values are about 7% and 4% above the critical values listed in table 1, showing that the simple mean-field theory is surprisingly accurate in this case. It is also worth noting that our results for the critical exponents are quite close to the values $\nu_\perp = 2/d, z = 2$ and $\beta = 1$ values provided by field theory [10] for the opposite case, $D_A < D_B$.

As noted in the introduction, the continuous-transition scenario seems quite reasonable in the DEP with $D_A > D_B$, as in the other two cases (equal diffusion rates and $D_A < D_B$). Although the present results accord perfectly with intuition, they are of course in sharp conflict with previous renormalization group analyses [10, 12]. While one might argue that the discontinuity is so weak as not to be observable in systems of the size studied here, we note that the estimate, equation (1), for the discontinuity $\Delta\rho_B$ is not intrinsically small. Thus the present results, and those of [11, 16], suggest some fundamental limitation of ϵ-expansion RG analysis applied to the diffusive epidemic process.

The present work is limited to the case $D_A > D_B$, which has been the most controversial. We defer for future work the other cases ($D_A = D_B$ and $D_A < D_B$), which should also be studied in two or more dimensions, to verify the three universality-class scenario as well as the detailed predictions of RG analysis. A further interesting subject for future work is a site-restricted ('fermionic') diffusive epidemic process.

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